# Time-on-Distance versus Time-on-Time PHRF Handicapping on Lake St. Clair[1]

Understanding the differences between time-on-distance and time-on-time handicapping is easier than usually appreciated. We will drop most of the historical baggage and approach this from a fresh perspective.

### Variable Name Conventions

Let's state some variable name conventions — we will use $t$ and $\check{\check{t}}$ (with two *checks* on top) for elapsed and corrected times in seconds, $d$ for course length in nautical miles and $p$ and $\check{\check{p}}$ for *pace* and *corrected pace* in seconds-per-mile, respectively

$$p = \frac{t}{d} \qquad \check{\check{p}} = \frac{\check{\check{t}}}{d}$$

Pace is a measure of how long it takes to complete a distance; it is a reciprocal to speed $v$ measured in knots. To convert from seconds-per-mile to knots or vice-versa

$$v = \frac{3600\,{}^{\mathrm{s}}\!/\!_{\mathrm{hr}}}{p} \qquad p = \frac{3600\,{}^{\mathrm{s}}\!/\!_{\mathrm{hr}}}{v}$$

A *general purpose handicap* (abbreviated GPH and denoted by a variable $g$) is a pace that represents a boat's average performance — i.e. it is a standard pace that aggregates performance over an entire course and across different wind ranges. A star ★ indicates the scratch boat for a fleet and ${}^{\bigstar}g$ its handicap.

### Handicaps

To determine how well boats perform in a race we order them by how well each performs relative to its own standard pace, $p - g$ for time-on-distance and ${}^{p}\!/\!_{g}$ for time-on-time. Corrected paces are calculated

$$\begin{aligned} \check{\check{p}} &= p + {}^{\bigstar}g - g && \text{for time-on-distance} \\ \check{\check{p}} &= p \times \tfrac{{}^{\bigstar}g}{g} && \text{for time-on-time} \end{aligned}$$

The *scratch handicap* ${}^{\bigstar}g$, which is common for all boats, does not effect the handicapped finish order and is used to present corrected paces in a convenient form. Using the handicap of a boat in the race as ${}^{\bigstar}g$ is best for a side-by-side comparison of time-on-distance to time-on-time — for the scratch boat the time-on-distance and time-on-time corrected pace will be the same as its observed pace $\check{\check{p}} = p$ (and likewise for corrected and elapsed times $\check{\check{t}} = t$). It is conventional to choose the fastest handicapped boat as the scratch boat, but this is not necessary; any boat can be chosen.

Calculating corrected pace directly from the observed pace $p = {}^{t}\!/\!_{d}$ is the most natural way to apply handicaps, but it is more conventional to multiply out by course distance to calculate corrected times from elapsed times

$$\begin{aligned} \check{\check{t}} &= t + ({}^{\bigstar}g - g)d &&= t + {}^{\bigstar}gd - gd && \text{for time-on-distance} \\ \check{\check{t}} &= t \times \tfrac{{}^{\bigstar}g}{g} &&= t \times \tfrac{{}^{\bigstar}gd}{gd} && \text{for time-on-time} \end{aligned}$$

When multiplying a handicap by course distance to get an interval of time we will call the product a *course specific handicap*.

[1]This document is adjunct to the *ToD to ToT Handicapping Comparison* on the web site *www.southportsailingclub.com/Results/handicappingComparison.html*

### How to Apply Time-on-Time Handicapping on the Race Course

Consider the *time allowance* $\Delta t$ as the difference in time between a pair of boats that shall correct out the same and $\Delta g$ as the corresponding difference in handicaps. Then these must satisfy the proportionality

$$\Delta t \ : \ \Delta g \quad = \quad t \ : \ g$$

the ratio of the time-on-time time allowance $\Delta t$ to the difference in handicap $\Delta g$ is equal in proportion to the ratio of elapsed time $t$ to the handicap $g$. From this we can derive a simple formula

$$\Delta t \quad = \quad \frac{\Delta g}{g} \times t$$

but it is often easier to use the proportionality directly.

> Let's work an example — the other boat has a 790 s/mi handicap — you are the faster boat with a 765 s/mi handicap — then for every 765 s = 12 min 45 s of elapsed time you must gain 790 s − 765 s = 25 s on your competitor. If you finish with an elapsed time of 1 h 30 min which is approximately 7 × 12 min 45 s then you need to win by approximately 7 × 25 s = 2 min 55 s.
>
> Note that you can calculate time allowances for all the boats you are racing against by adding the $\Delta g$ appropriate for each boat every 765 s = 12 min 45 s of your own elapsed time. The time interval is only dependent on your own handicap making it easy to track all your competitors simultaneously with only a little bit of preparation.

### Time Allowances for Time-on-Distance versus Time-on-Time Handicapping

Consider the time allowances for time-on-distance and time-on-time in comparison to each other. The time allowance for time-on-distance $\Delta t = \Delta gd$ (the difference in course specific handicaps) does not change throughout the race whereas time allowance for time-on-time increases proportionally throughout. As proportionalities in comparable terms this is expressed

$$\begin{array}{lll} \Delta t \ : \ \Delta gd & = \ 1 \ : \ 1 & \text{for time-on-distance} \\ \Delta t \ : \ \Delta gd & = \ t \ : \ gd & \text{for time-on-time} \end{array}$$

the ratio of the time allowance $\Delta t$ to the difference in course specific handicap $\Delta gd$ is one to one for time-on-distance and is equal in proportion to the ratio of elapsed time to course specific handicap for time-on-time. The ratio on the right can refer to either boat (for purposes of argument consider it your own boat). Whenever your elapsed time $t = gd$ then the time allowance is identical whether using time-on-distance or time-on-time handicapping. When $t < gd$ (i.e. your pace is faster than your standard pace) then the time allowance is less using time-on-time then it would be for time-on-distance handicapping. When $t > gd$ (i.e. your pace is slower than your standard pace) then the time allowance is greater using time-on-time then it would be for time-on-distance handicapping.

### Which is Better?

From the presentation above, it would seem time-on-distance or time-on-time handicapping can be used interchangeably — after all, sometimes a boat will sail faster than its average pace and sometimes slower. The choice would seem to come down to which better models the actual relative performance of boats in varying conditions. The overwhelming consensus, supported by the velocity prediction programmes used for measurement handicaps, is that time-on-time handicapping can do the job better than time-on-distance, the historical basis for PHRF handicapping.

There is a complication which makes the use of the same handicap for both time-on-distance and time-on-time difficult. For a boat to have a standard pace it really needs to sail in standard conditions — and average conditions clearly differ from place to place. It would seem that the best method for localizing such a system of handicaps is to honour the time-on-time model and scale all the standard paces by a common factor to reflect the standard conditions for the place. Note the ordering of corrected times using time-on-time handicaps are unaffected by such a transformation — time allowances for (the less representative) time-on-distance handicaps would be scaled appropriately for the average conditions. Localizing a system of handicaps by simply shifting by a common offset would preserve the ordering of corrected times using time-on-distance handicapping but is known to be less accurate than scaling.

Now as long as either time-on-distance or time-on-time handicapping is used consistently only the relative performance of boats is relevant to either the computation or application of handicaps — this can obscure the underlying dependence on average conditions when using both — and obscure the systemic weakness of time-on-distance handicaps that are not properly localized.

### The Relative Gauge

Within the regime of time-on-distance PHRF handicapping across North America the focus on relative performance means that the expected differences in absolute paces have been largely ignored, with little or no attempt to reconstruct absolute paces until the recent shift to time-on-time handicapping. Traditional PHRF numbers, which were meant to be used only for time-on-distance handicapping, have been defined relative to a boat with a specified zero PHRF rating.

We add $557\,{}^{s}/_{mi}$ to PHRF numbers to reconstruct a standard pace and general purpose handicap $g$. As long as you restrict your attention to time-on-distance handicapping the shifted and unshifted gauges behave identically. The $557\,{}^{s}/_{mi}$ stated above is not part of the traditional definition of PHRF numbers but a modern reconstruction of the absolute pace implicit in the use of time-on-distance handicapping. In standard conditions a zero PHRF rated boat is expected to travel at an average pace of $557\,{}^{s}/_{mi}$.

### Whoa

But there is a huge catch here. For boats on Lake St. Clair your calculated standard pace $g = \text{PHRF} + 557\,{}^{s}/_{mi}$ will be faster than can be reasonably expected and resulting time allowances between boats using time-on-time handicapping will be greater than for time-on-distance more often than not. So the game really does change, though whether that is enough to alter the final ranking of finishes depends on how close the racing really is. The justification for this seems to be that our local time-on-distance handicaps are faulty, they show systemic bias inherent to using ratings from other stations which have greater winds and less difference in their expected paces. WYC and all the American clubs on the lake accept this and have used the $g = \text{DRYA PHRF} + 557\,{}^{s}/_{mi}$ conversion for time-on-time handicapping since 2007.

The counter argument is that the DRYA and ECPHRF rating stations have already adapted their ratings over the years to properly account for local conditions using time-on-distance handicapping and therefore the $+557\,{}^{s}/_{mi}$ offset is too small for Lake St. Clair.

### Over-correcting

Lake Ontario PHRF doesn't even try to reconstruct a realistic standard pace and uses an unrealistically small (and peculiarly precise) $+401.431\,\text{s/mi}$ offset to compensate for what they consider poor time-on-distance handicaps. While it is certainly easy to incorporate such adjustment factors into a corrected time formula — being no more than tweaking a single number — it does undercut the physical interpretation of such handicaps and lead to wildly different results using time-on-distance versus time-on-time handicapping. Some would get around this by interpreting the PHRF handicap in s/km and the offset as $+401.431\,\text{s/km}$ for the general purpose handicap.

### Other Possible Reconstructions

Given the uncertainty in the $+557\,\text{s/mi}$ reconstruction it would perhaps have been better to have chosen an offset more appropriate for mental arithmetic $+600\,\text{s/mi} = +10\,\text{min/mi}$. This might even have the virtue of honouring the average performance on Lake St. Clair. At a local level, it is better to explicitly correct for any historical bias in each boat's rating than to implicitly re-rate boats and silently ignore the systemic differences between time-on-time and time-on-distance handicapping.

Unfortunately, DRYA has already chosen the $+557\,\text{s/mi}$ offset. And, given the push for time-on-time handicapping, perhaps it is better to abandon time-on-distance as no longer being relevant to local PHRF racing.

### Corrected Time Formulas in the PHRF Gauge

Using a zero PHRF rated boat as scratch the time-on-distance formula for corrected times takes on a particularly simple form with the balancing ★ term eliminated

$$\check{\check{t}} = t - \text{PHRF} \times d$$

This choice of gauge ensured that corrected times were easy to calculate by hand using only positive numbers but is now annoying. It would be more convenient for competitors to have the standard paces published.

And a zero PHRF rated boat is rarely a good choice for a scratch boat. Letting ★PHRF be the PHRF number for a scratch boat the corrected time formulas need to be tweaked to accommodate the shifted gauge

$$\begin{aligned} \check{\check{t}} &= t + ({}^{\star}\text{PHRF} - \text{PHRF}) \times d && \text{for time-on-distance} \\ \check{\check{t}} &= t \times \frac{{}^{\star}\text{PHRF} + 557\,\text{s/mi}}{\text{PHRF} + 557\,\text{s/mi}} && \text{for time-on-time} \end{aligned}$$

These are the corrected time formulas after having substituted for $g = \text{PHRF} + 557\,\text{s/mi}$ and ${}^{\star}g = {}^{\star}\text{PHRF} + 557\,\text{s/mi}$ and then having simplified. The transformed formulas are not much more complicated than those that are defined in terms of a general purpose handicap but they are inelegant and their physical significance is obscured.

### Dead Weight

PHRF numbers carry a lot of historical dead weight. For time-on-time handicapping we would be better served to have ratings with the $+557\,\text{s/mi}$ shift already incorporated. And for time-on-distance handicapping these numbers should be scaled to better reflect expected paces.

## More Dead Weight

For time-on-time handicapping, another historical albatross is the publication and use of *time correction factors*

$$\text{TCF} = \frac{{}^{\star}\text{PHRF}{+}557\,\text{s/mi}}{\text{PHRF}{+}557\,\text{s/mi}}$$

These are even more awkward to deal with than the underlying PHRF numbers as they must be inverted before being used in a proportionality to determine time allowances. Now the inverse itself

$$\frac{\text{PHRF}{+}557\,\text{s/mi}}{{}^{\star}\text{PHRF}{+}557\,\text{s/mi}}$$

is convenient for working with time allowances and, when expressed as a decimal fraction or a percentage, may be considered a useful analogue to a Portsmouth handicap. Whichever form is used, the use of a single scratch handicap for all divisions will result in obscure corrected times for most boats.

## Summary

In summary, these are the corrected time formulas in common use after reconstructing a meaningful standard pace and general purpose handicap $g = \text{PHRF} + 557\,\text{s/mi}$

**In conventional ECPHRF form for time-on-distance** with a scratch ${}^{\star}\text{PHRF} = 0\,\text{s/mi}$

$$\begin{array}{llcccc} \text{time-on-distance} & \check{\check{\check{t}}} & = & t-(g-557\,\text{s/mi})\times d & = & t-\text{PHRF}\times d \\ \text{time-on-time} & \check{\check{\check{t}}} & = & t\times\frac{557\,\text{s/mi}}{g} & = & t\times\frac{557\,\text{s/mi}}{\text{PHRF}{+}557\,\text{s/mi}} \end{array}$$

**In conventional DRYA form for time-on-time** with a scratch ${}^{\star}\text{PHRF} = 93\,\text{s/mi}$

$$\begin{array}{llcccc} \text{time-on-distance} & \check{\check{\check{t}}} & = & t-(g-650\,\text{s/mi})\times d & = & t-(\text{PHRF}-93\,\text{s/mi})\times d \\ \text{time-on-time} & \check{\check{\check{t}}} & = & t\times\frac{650\,\text{s/mi}}{g} & = & t\times\frac{650\,\text{s/mi}}{\text{PHRF}{+}557\,\text{s/mi}} \end{array}$$

**In general form** with an arbitrary scratch ${}^{\star}g = {}^{\star}\text{PHRF} + 557\,\text{s/mi}$

$$\begin{array}{llcccc} \text{time-on-distance} & \check{\check{\check{t}}} & = & t-(g-{}^{\star}g)\times d & = & t-(\text{PHRF}-{}^{\star}\text{PHRF})\times d \\ \text{time-on-time} & \check{\check{\check{t}}} & = & t\times\frac{{}^{\star}g}{g} & = & t\times\frac{{}^{\star}\text{PHRF}{+}557\,\text{s/mi}}{\text{PHRF}{+}557\,\text{s/mi}} \end{array}$$

**For time-on-distance the time allowances $\Delta t$** for boats to correct out to the same are

$$\Delta t \quad = \quad \Delta g\times d \quad = \quad \Delta\text{PHRF}\times d$$

**And for time-on-time the time allowances $\Delta t$** are determined by the proportionalities

$$\begin{array}{rcl} \Delta t : \Delta g & = & t : g \\ \Delta t : \Delta\text{PHRF} & = & t : \text{PHRF} + 557\,\text{s/mi} \end{array}$$